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★**Introduction to graph and hypergraph theory.**

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From the preface: "Graph theory is an important area of contemporary mathematics with many applications in computer science, genetics, chemistry, engineering, industry, business and social sciences. It is a young science invented and developed for solving challenging problems of 'computerized' society for which traditional areas of mathematics such as algebra or calculus are powerless.

"This book is for math and computer science majors, and for students and representatives of many other disciplines (like bioinformatics, for example) taking courses in graph theory, discrete mathematics, data structures, and algorithms. It is also for anyone who wants to understand the basics of graph theory, or just is curious. No previous knowledge in graph theory or any other significant mathematics is required. The very basic facts from set theory, proof techniques and algorithms are sufficient to understand it; but even those are explained in the text.

"Structurally, the text is divided into two parts, where Part II is the generalization of Part I. The first part discusses the key concepts of graph theory with emphasis on trees, bipartite graphs, cycles, chordal graphs, planar graphs and graph coloring. The second part considers generalizations of Part I and discusses hypertrees, bipartite hypergraphs, hypercycles, chordal hypergraphs, planar hypergraphs and hypergraph coloring. There is an interaction between the parts and within the parts to show how ideas of generalizations work. The main point is to exhibit the ways of generalizations and interactions of mathematical concepts from the very simple to the most advanced.

"Hypergraphs model practical situations in different sciences in a much more general setting than graphs do. In addition, they help to find optimal solutions for many new optimization problems. While vertices represent the elements of a set (as in graphs), the hyperedges represent subsets of any cardinality (not just 2 as in graphs), or, even more generally, arbitrary statements about arbitrary subsets.

"One of the features of this text is the duality of hypergraphs. There are only two players in graph theory: vertices and edges. In dual hypergraphs, they just swap roles. This fundamental concept is missing in graph theory (and in its introductory teachings) because dual graphs are not properly graphs, and they generally represent hypergraphs. However, as Part II shows, duality is a very powerful tool in understanding, simplifying and unifying many combinatorial relations; it is basically a look at the same structure from the opposite (vertices versus edges) point of view. Teaching and applying graph theory without hypergraphs does not allow one to use duality; it is like teaching graphs without their complements. Among the goals of the text, one is to fill this gap.

"Part I may be used at the undergraduate level for a one semester introductory course, and Part II may be used as a text or supplement for senior and graduate students. Some chapters or sections from Part II may be used at the undergraduate level for the most advanced students as projects in undergraduate research to report on departmental seminars. The book includes many examples, figures and algorithms; each section ends with a set of exercises and a set of computer projects. The answers and hints to selected exercises are provided at the end of the book. The material has been tested in class during more than 20 years of teaching experience of the author. Math majors will pay more attention to theorems and proofs, computer science majors will work more with the concepts, algorithms and computations, and representatives of other sciences will find models and ideas for solutions of optimization problems in their fields.

“Regarding the contents, four core areas of graph theory have been chosen: bipartite graphs, chordal graphs, planar graphs and graph coloring. The text exhibits a survey of basic results and their generalizations to hypergraphs in these areas. Bipartite graphs, planar graphs and graph coloring were the source and the origin of graph theory. Chordal graphs, discovered much later, have a very special place in the entire theory: they are the best playground for introduction to graphs and hypergraphs. The fact is that many unrelated (!) fundamental parameters introduced for general graphs (like, for example, related to degrees, or complements, or colorings) achieve their optimal values on chordal graphs. There are many relations of chordal graphs to trees, but only the language of hypergraphs allows one to show that chordal conformal hypergraphs are dual to hypertrees. This is usually very impressive and unexpected to the reader since it is sufficient simply to transpose the incidence matrix of a hypertree to obtain a chordal hypergraph. It explicitly shows the strength of hypergraph theory.

“At last graph coloring, generalized to hypergraphs, allows one to consider the colorability, upper chromatic number, hypergraph perfection, the gaps in the chromatic spectrum, etc. Such concepts grew up from graph coloring and essentially represent the unfolding of graph coloring. Several basic results from mixed hypergraph coloring, taken, adapted and updated from [V. I. Voloshin, *Coloring mixed hypergraphs: theory, algorithms and applications*, Amer. Math. Soc., Providence, RI, 2002; MR1912135 (2003i:05058)], lead to unforeseen discoveries in Chapter 10; they demonstrate the power of generalizations. All this reflects the fact that for the last two decades, a significant number of new fundamental ideas, results and publications have led to the situation where hypergraph theory in general, and hypergraph coloring in particular, are taking a new shape. The theory has a great future since it continues to generate new research problems that never arose before.”