SAE TECHNICAL PAPER SERIES

932460

The System Theory Approach to Some Engineering Problems

Arcadii Z. Grinshpan
University of South Florida and Russian Academy of Sciences, INENCO

Sergey V. Belyi
University of South Florida

1993 International Off-Highway & Powerplant Congress & Exposition
Milwaukee, Wisconsin
September 13-15, 1993

SAE *The Engineering Society For Advancing Mobility Land Sea Air and Space*®
INTERNATIONAL

400 Commonwealth Drive, Warrendale, PA 15096-0001 U.S.A. Tel: (412)776-4841 Fax:(412)776-5760

ISSN 0148-7191

Printed in USA 90-1203B/PG

932460

The System Theory Approach to Some Engineering Problems

Arcadii Z. Grinshpan
University of South Florida and Russian Academy of Sciences, INENCO

Sergey V. Belyi
University of South Florida

ABSTRACT

The System Theory considers every engineering problem as a corresponding mathematical model with input and output spaces and usually with a controller. By the same token, it is presumed that an appropriate model has been constructed. It is known that methods applied in this theory are effective for linear or naturally linearizable mathematical models. This paper is designed to convince the reader that a mathematical background in System Theory for input-output models is helpful for a wider class of technical problems.

Also it seems to be important for development and approbation of System Theory methods to apply them to some new admissible objects. It turns out, that the basic ideas of System Theory can be applied to complicated physical optimizational problems connected with heat and mass transfer (with working process of diesel engines in particular). Actually certain procedures, which are typical for this mathematical theory, together with some other ones allowed us to reduce the central variational problem for "Diesel Engine - Fuel Injection Equipment" (DE/FIE) system to the finite dimensional optimizational problem for the corresponding parametric mathematical model. The main attributes of the System Theory approach and their construction for a DE/FIE system are described.

PREFACE

In writing this paper we tried to describe and illustrate some general approach for using modern mathematical methods (especially ones of the Functional Analysis and the Function Theory) to investigate some non-trivial engineering optimizational problems. Our purpose is to help some engineers overcome distrust of mathematics and its techniques and also to help technology development.

Very often creation of a comprehensive calculation method for a technical object is the main goal of related mathematical - engineering researches, because it allows to solve practical tasks of improvement of main quantitative characteristics of the object through perfecting some parameters. Reasonable realization of this goal permits to fulfil the following tasks when developing and modernizing a machine:

1) to shift the emphasis from real machine testing to computer-assisted calculations;

2) to solve some problems of optimum design of the corresponding machine on the basis of these calculations.

Let us assume that we have to deal with some complicated physical problem connected with heat and mass transfer for a technical object. We come from the point that all-round investigation of corresponding physical processes have already given a sufficient background for mathematical description of the considering object. That means we know what should be given to obtain a calculated-theoretical result applying elaborated model and a computer. Roughly speaking, some calculation method is already created. We assume also that the main principles used for guidance are as follows:

1. In spite of the vastness of the problem in-

curred, the calculation method should comprise (in the main features) the complete object as far as possible. Only such an approach gives a firm foundation for solving practical optimizational tasks.

2. Mathematical model of the technical object should include both commonly known laws of physics (such as laws of conservation of energy, impulse, mass, etc.) and, perhaps many of the empirical rules and observations accumulated over a long time by specialists in the corresponding engineering fields.

3. An effective mathematical apparatus for solving some complex and non-standard problem cannot be standard as a rule. The usual approach based on compiling mathematical standard procedures, which is widely used by research engineers, is not so effective. To reduce the task to a variational or an optimizational problem and to ensure the realization of a particular character of the model it is useful to create a complete mathematical apparatus specifically suited for the considering task.

4. The mathematical model on the basis of which the method is realized must not constitute a dogmatic construction. On the contrary, its continuous updating in accordance with the results of new researches should be ensured.

5. The main goal of such kind of researches is to be able to solve real practical tasks, that is a continuous application of the method in practice.

Of course, real complicated heat/mass transfer problems have several aspects of importance: technical and design, ecological, physical, mathematical, that of computer techniques and programming. However, now we are interested in the development of the mathematical side only. Creating of a comprehensive mathematical apparatus for calculations is an important but not final step connected with application of mathematics to considered technical object. Some reorientation of this apparatus towards powerful methods of abstract mathematics using the System Theory approach can be the next step. We hope that this will increase the efficiency of mathematical-engineering researches and make range of problems which could be investigated wider.

THE SYSTEM THEORY SUBJECT

In the recent times the System Theory has developed into a scientific and engineering discipline which seems destined to have an impact upon all aspects of modern society [1,2]. First studied as Control Theory by mathematicians and engineers, System Theory interest is now increasing in the different branches of science. System Theory is a body of concepts and techniques which are used to analyze and design technical systems of various types (for example, [3]), regardless of their special physical natures and functions. The language of System Theory is mathematics and any serious attempt of using System Theory must be accompanied by the acquisition of mathematical precision and understanding. To be precise System Theory language is a language of functional and vector spaces and vector-valued functions.

We may consider any technical system as a "black box" into which we feed inputs, and out of which we receive outputs. In these talks the inputs will be, in general, vector-valued functions, usually of time, into the input space $\mathbf{E}$ and the outputs are vector-valued functions into the output space $\mathbf{W}$.

Common physical properties of a "box" are: causality (if two inputs are identical then the outputs are identical too), time invariance (an experiment at noon gives the same answer as an experiment at midnight), stability (every decaying input function produces a decaying output function). Conceptually the "box" is what is given. It acts on some space of functions which we chose to put in the "box" and the responds by putting out other functions. In order to specify how the inputs determine the outputs we must add a description of some internal state of the considered technical system. Let $\mathbf{X}$ be the set of internal states of the system, we will call $\mathbf{X}$ a state space. We stick to single-input single-output feedback control system (SISO in literature). The most elementary feedback control system has three components: a plant (the object to be controlled), a sensor (to measure the output of plant), and a controller. In such kind of system the output is processed and information from it is fed back into the input to control the performance. The role of the controller is to correct and generate the plant's input. Generally speaking, the objective in a control system is to make some output behave in a desired way by manipulating some input. The simplest objective might be to keep output close to some equilibrium point.

The introduction of norms [4] for input and output spaces is necessary for measurement of data and evaluating of current results. Which norm is appropriate depends on the situation at hand. Let us recall that a norm must have the following three properties for all **u** and **v** from the corresponding normed space:

1) $\|\mathbf{u}\| \geq 0, \|\mathbf{u}\| = 0$ if and only if $\mathbf{u} = \mathbf{0}$

2) $\|\lambda\mathbf{u}\| = |\lambda| \cdot \|\mathbf{u}\|$, for all real λ

3) $\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$

Mathematically the behavior of the system can be described in the terms of some operators with related domains in the appropriate spaces [4]. The mapping that maps input of the certain system into the output is called transfer operator (or transfer function) of this system. One of the most important direct problems of System Theory is so called OPT (optimizational) problem of obtaining a desired output.

CENTRAL DIESEL ENGINE - FUEL INJECTION EQUIPMENT VARIATIONAL PROBLEM

In [5], there is some description of years-long researches which main goal is creating of a comprehensive calculation method for the "Diesel Engine - Fuel Injection Equipment" (DE/FIE) system. Full realization of this goal allows to solve practical tasks of improvement of the main quantitative characteristics of diesel engines through perfecting their fuel injection equipment. Briefly it was discussed in the talk of A.Grinshpan and S.Romanov at the International Off-Highway and Powerplant Congress (Milwaukee, WI, September 9-12, 1991). We repeat now the main ideas since it is necessary for proper understanding of construction of main System Theory attributes for DE/FIE system.

It was chosen for an object a diesel engine with direct injection (DI) and with mainly volume or volume/boundary mixture formation process, as this type of engines is widespread at present and has the bright prospects for the future.

As a result of the analysis of the present-day practices, the following technical task have been formulated, the solution of which should be ensured by the method proposed: It is obtaining the best fuel economy possible for a specific engine with known design, setting and duty parameters within the pre-determined limits of standards for emissions, knocking level and service life of the engine and its fuel injection equipment.

Fuel economy is determined by the value cycle work minus losses in the fuel injection drive; knocking level - by the pressure-rise rote; engine service life - by the allowed value of combustion pressure and service life of fuel injection equipment - by the allowed value of contact stresses in cam drive.

Fuel injection equipment affects diesel engine working process through its output characteristics: injection pressure characteristic, effective cross-section of nozzle holes, number of nozzle holes and their positions relative to each other and combustion chamber, actual advance angle. Therefore, determination of output parameters of the fuel delivery process which ensure the best fuel economy within the limitations imposed is the first necessary step towards solving the problem. The second natural step is determination of FIE design parameters that ensure gaining the necessary output parameters of fuel delivery process.

Physical aspect of the work concerns analysis of the processes through which the working process of the DE/FIE system is realized, finding out the critical ones and creating their respective physical models that make it possible to solve the problem set.

In this problem it follows that the proposed method should encompass the groups of physical processes as described below:

- physical processes in the engine cylinder (compression, fuel spray development, heat and mass transfer between atomized fuel and its environment, mixture formation, ignition, combustion, expansion);

- physical processes within the fuel injection equipment itself (pressure impulse generation and propagation along high pressure line, its transformation into velocity ram of the fuel spray).

As previously stated the method is meant to deal with DI diesel engines with mainly volume or volume/boundary mixture formation process. Therefore the natural basis for calculations should be a worked-out physical model of the spray. It has been developed [6,5] a phenomenological model of such a spray based on the experimental data on its structure and mixture formation mechanism. An essential point of the model is that it takes into account the role of high concentration of liquid particles in the spray which it plays in the processes of impulse exchange and heat and mass transfer. Calculations of the heat and mass transfer processes are based on Ranz-Marschall expressions and take into account real properties of liquids in a wide range of pressures and temperatures. It has been also developed [5] a specific phenomenological model of diesel fuel oil that allows to take into account the influence of a complex multi-fraction composition of the fuel on the heat and mass exchange processes.

From the mathematical point of view it was essential to create a mathematical apparatus and (on its basis) to solve an original variational problem with free ends and additional functional limitations. Reduced indicated efficiency of the DE/FIE system or its useful work are used as a rule, as the estimated main functional. Injection pressure characteristic is the variable function with a pre-determined integral (cycle fuel delivery). Apart from that, there is a variable vector. Its components as follows: crank angles of the injection beginning and ending, number of nozzle holes, effective area of nozzle holes and other ones. Functional limitations are caused by limitations on the part of ecology, knocking, engine service life and design parameters of the FIE.

THE CONSTRUCTION OF MAIN SYSTEM THEORY ATTRIBUTES FOR DE/FIE SYSTEM

We interpret the present problem on language of functional and vector spaces and vector-valued functions. Let $\mathbf{R}^m$ be m-dimensional Euclidean space of real vectors $\mathbf{x} = (x_1, ..., x_m)$ with corresponding norm

$$\|\mathbf{x}\|_{\mathbf{R}^m} = (x_1^2 + .. + x_m^2)^{1/2}.$$

We define operation of convolution for a fixed nonzero vector $\mathbf{a} \in \mathbf{R}^m$ as follows

$$\mathbf{a} * \mathbf{u} = (a_1 u_1, ..., a_m u_m)$$

for all vectors $\mathbf{u} = (u_1, ..., u_m) \in \mathbf{R}^m$. We obtain

$$\|\mathbf{a}*\mathbf{u}\|_{\mathbf{R}^m} = (y_1^2 + ... + y_m^2)^{1/2},$$

where $y_k = a_k u_k (k = 1, ..., m)$.

As usually we denote by $\mathbf{C}_{[\alpha,\beta]}$ the class of all continuous functions $f(\phi)$ on the interval $[\alpha, \beta]$. Let $\|f\|_{\mathbf{C}_{[0,2\pi]}}$ be some norm in $\mathbf{C}_{[0,2\pi]}$. There are several appropriate choices of norms in this class. For example, we can indicate two of them:

1. $\|f\|_{\mathbf{C}_{[0,2\pi]}} = \sup_{[0,2\pi]} |f(\phi)w(\phi)|$, where $w(\phi)$ is some weight function in the interval $[0, 2\pi]$.

2. $\|f\|_{\mathbf{C}_{[0,2\pi]}} = \left(\int_0^{2\pi} |f(\phi)|^2 d\mu(\phi)\right)^{1/2}$, where $\mu(\phi)$ is a positive bounded measure.

In our case the input space $\mathbf{E}$ is the space of $\mathbf{R}^{m+1}$-valued functions $\mathbf{f} = (f, \mathbf{u})$, $\mathbf{E}$ coincides with the Cartesian product $\mathbf{C}_{[0,2\pi]} \otimes \mathbf{R}^m$. For $\mathbf{f} \in \mathbf{E}$ we define

$$\|\mathbf{f}\|_E = (\|f\|^2_{\mathbf{C}_{[0,2\pi]}} + \|\mathbf{a} * \mathbf{u}\|^2_{\mathbf{R}^m})^{1/2}.$$

It is not difficult to check three basic properties of norm.

However not all elements of $\mathbf{E}$ are admissible for us. We deal only with finitary functions from the class $\mathbf{C}_{[0,2\pi]}$. It is well known that a function $f(\phi) \in \mathbf{C}_{[0,2\pi]}$ is finitary if there exist such $\epsilon_1, \epsilon_2 > 0$, $\epsilon_1 + \epsilon_2 < 2\pi$, that $f(\phi) = 0$ for all values of ϕ from the set $[0, \epsilon_1] \cup [2\pi - \epsilon_2, 2\pi]$. Moreover, finitary functions we consider are so called one-model. That means that any such a function has only one local extremum. In fact this extremum is a global maximum of this function.

Vectors $\mathbf{u} = (u_1, ..., u_m)$ are used as so called vectors with variable parameters for problem considered in previous section. In particular, $u_1 = \phi_{inj.b.}$ (crank angle of the injection beginning), $u_2 = \phi_{inj.e.}$ (crank angle of the injection ending), $u_3 = i_{n.h.}$ (number of nozzle holes), $u_4 = \mu f_{n.h.}$ (effective area of nozzle holes), etc. Clearly every entry of $\mathbf{u}$ varies in certain interval. Therefore we can consider vectors $\mathbf{u}$ only in some parallelipiped $\mathbf{\Pi} \in \mathbf{R}^m$ which sides are connected with restrictions on related components.

Let $\mathbf{P}(g_c)$ be the class of theoretical injection pressures with fixed cycle fuel delivery g_c and two-sided restrictions for second derivative (m and M are lower and upper bounds respectively).

The class $\mathbf{P}(g_c)$ contains of twice differentiable, finitary, one-model functions $p(\phi) \in \mathbf{C}_{[0,2\pi]}$ with

$$k_1 \cdot \mu f_{n.h.} \int_0^{2\pi} p(\phi) d\phi = g_c$$

(k_1 is a coefficient of proportionality),

$$m < p''(\phi) < M,$$

and $p(\phi) = 0$ if $\phi \in [0, \phi_{inj.b.}] \cup [\phi_{inj.e.}, 2\pi]$.

Now we are able to define the admissible set $\mathbf{D}$ from $\mathbf{E}$ with the norm $\|\cdot\|_{\mathbf{E}}$

$$\mathbf{D} = \{\mathbf{f} : \mathbf{f} = (p, \mathbf{u}) \in \mathbf{E}, \mathbf{u} \in \mathbf{\Pi}, p \in \mathbf{P}(g_c)\}.$$

It appears to be difficult to solve the considering variational problem on the whole admissible set $\mathbf{D}$. Therefore we construct some $n+m$-parametric set $\mathbf{D}_{n+m}$ of vector-functions from $\mathbf{D}$. For this purpose we replace class $\mathbf{P}(g_c)$ in the definition of $\mathbf{D}$ by n-parametric family $\mathbf{P}_n(g_c)$ of functions $p \in \mathbf{P}(g_c)$. There are several possible ways to obtain such kind of a reasonable parameterization (see [7]). Resulted $n+m$-parametric set $\mathbf{D}_{n+m}$ is isomorphic to some set $\mathbf{B}_{n+m}$ in the $\mathbf{R}^{n+m}$. In our case the state space $\mathbf{X}$ contains $\mathbf{D}_{n+m}$. Thus the central variational problem for DE/FIE system is reduced to the $n+m$-dimensional optimizational problem for the restriction of our mathematical model onto the set $\mathbf{D}_{n+m}$.

The above mentioned procedures and consequent ones, which are necessary for solving the task, are connected with introduction of some operators on the admissible set and on the parametric set $\mathbf{D}_{n+m}$. These operators are the most important part of structure of our model system. Usually, in the System Theory the action of the model is interpretered as main operator. In our case main operator is defined on the set $\mathbf{D}_{n+m}$ (or on the whole set $\mathbf{D}$). Transfer operator in this setting is appropriate composition of main and auxiliary operators. Furthermore the following types of other operators appear:

1. "close to identity" ($\mathbf{D}_{n+m} \to \mathbf{D}_{n+m}$ or $\mathbf{B}_{n+m} \to \mathbf{B}_{n+m}$);
2. operators of one-component linearization ($\mathbf{D} \to \mathbf{D}_{n+m}$);
3. norm preserving operators of vectorization ($\mathbf{D} \to \mathbf{B}_{n+m}$);
4. correction operators ($\mathbf{D}_{n+m} \to \mathbf{D}_{n+m}$);
5. "jump"-operators ($\mathbf{D}_{n+m} \to \mathbf{D}_{n+m}$ or $\mathbf{B}_{n+m} \to \mathbf{B}_{n+m}$), etc.

Full description of all required operators, main functional (see below) and other components of our model system would make this paper large and too complicated mathematically.

The output space $\mathbf{W}$ contains $\mathbf{R}^{l+\nu+2}$-valued functions of the form

$$\mathbf{g} = (A, \mathbf{q}, \mathbf{e}).$$

Here $\mathbf{e} = (\mathbf{d}, h)$ is $\mathbf{R}^{l+1}$-valued FIE design parametric function, where $\mathbf{d} \in \mathbf{R}^l$ is FIE design parametric vector, $h = h(\phi)$ is a cam profile, which is from the class $\mathbf{H}$ of twice differentiable functions on the interval $[0, \pi]$ with some restrictions on the function and derivatives values.

A is the main functional, which is useful work of the DE/FIE system (or we could take its reduced indicated efficiency)

$$A = k_2 \cdot \int_0^{2\pi} N(\phi, p(\phi), p'(\phi), \mathbf{u}) d\phi,$$

where k_2 is a coefficient of proportionality,

$$\mathbf{u} = (\phi_{inj.b.}, \phi_{inj.e.}, i_{n.h.}, \mu f_{n.h.}, ...) \in \mathbf{\Pi},$$

$$p(\phi) \in \mathbf{P}(g_c) \text{ (or } \mathbf{P}_n(g_c)),$$

N is instantaneous power

$$N = \begin{cases} P_c V_c', & \text{if } \phi \in [0, \phi_{s.b.}] \cup [\phi_{m.p.e.}, 2\pi] \\ P_c V_c' - P_{o.p.} V_{o.p.}', & \text{if } \phi \in [\phi_{s.b.}, \phi_{m.p.e.}] \end{cases}$$

P_c is cylinder pressure, V_c is cylinder swept volume, $P_{o.p.}$ is over-plunger pressure, $V_{o.p.}$ is over plunger volume, $\phi_{s.b.}$ is plunger stroke beginning, $\phi_{m.p.e.}$ is crank angle of the maximum plunger elevation.

$\mathbf{q} \in \mathbf{R}^\nu$ is vector which entries are values of functionals that characterize the fulfillment of limitations of ecology, knocking, engine service life and design parameters of the FIE. It is always possible to put limitations such that all these entries are nonnegative, i.e. $\mathbf{q} \in \mathbf{R}_+^\nu$.

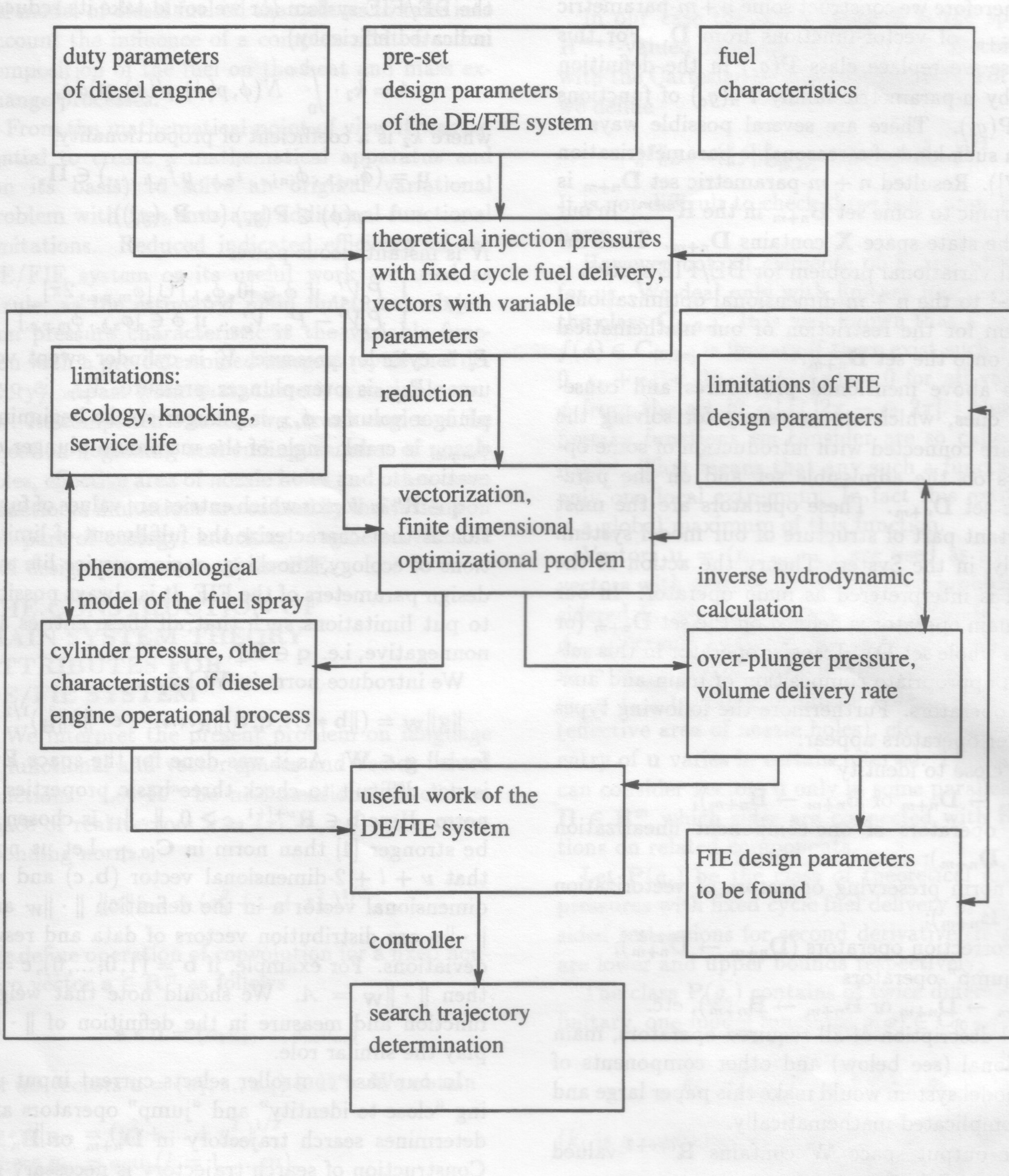
We introduce norm in $\mathbf{W}$ by

$$\|\mathbf{g}\|_{\mathbf{W}} = (\|\mathbf{b} * (A, \mathbf{q}, \mathbf{d})\|_{\mathbf{R}^{\nu+l+1}} + c \cdot \|h\|_{\mathbf{H}}^2)^{1/2}$$

for all $\mathbf{g} \in \mathbf{W}$. As it was done for the space $\mathbf{E}$ it is not difficult to check three basic properties of norm. Here $\mathbf{b} \in \mathbf{R}^{\nu+l+1}, c \geq 0$, $\|\cdot\|_{\mathbf{H}}$ is chosen to be stronger [4] than norm in $\mathbf{C}_{[0,\pi]}$. Let us note that $\nu + l + 2$-dimensional vector $(\mathbf{b}, c)$ and m-dimensional vector $\mathbf{a}$ in the definition $\|\cdot\|_W$ and $\|\cdot\|_{\mathbf{E}}$ are distribution vectors of data and result deviations. For example, if $\mathbf{b} = (1, 0, ..., 0), c = 0$ then $\|\cdot\|_{\mathbf{W}} = A$. We should note that weight function and measure in the definition of $\|\cdot\|_{\mathbf{E}}$ play the similar role.

In our case controller selects current input using "close to identity" and "jump" operators and determines search trajectory in $\mathbf{D}_{n+m}$ or $\mathbf{B}_{n+m}$. Construction of search trajectory is necessary for solving OPT problem directed to maximization of main functional A under the condition $\mathbf{q} \in \mathbf{R}_+^\nu$.

OUTLINE OF THE SOLUTION OF THE CENTRAL DE/FIE VARIATIONAL PROBLEM



Restatement of the DE/FIE problem using System Theory language allows the following:

1. The problem is more accessible for application of well known tools of abstract mathematics.

2. System Theory receives new stimulating object.

3. The sample for setting of such kind of problems (usually even simpler) is given as an object of System Theory.

NOMENCLATURE

A - useful work of the system

$\mathbf{a}$ - distribution vector of data deviation

$\mathbf{B}_{n+m}$ - isomorphic to $\mathbf{D}_{n+m}$ set from $\mathbf{R}^{n+m}$

$(\mathbf{b}, c)$ - distribution vector of result deviation

$\mathbf{C}_{[\alpha,\beta]}$ - class of all continuous functions on the interval $[\alpha, \beta]$

$\mathbf{D}$ - admissible set from $\mathbf{E}$

$\mathbf{D}_{n+m}$ - $n+m$-parametric representative set from $\mathbf{D}$

DE - diesel engine

DI - direct injection

$\mathbf{d}$ - FIE design parametric vector

$\mathbf{E}$ - input space

$\mathbf{e}$ - FIE design parametric function

FIE - fuel injection equipment

g_c - cycle fuel delivery

$\mathbf{H}$ - class contains of all admissible cam profiles

$h(\phi)$ - cam profile

$i_{n.h.}$ - number of nozzle holes

k_1, k_2 - coefficients of proportionality

N - instantaneous power

$\mathbf{P}(g_c)$ - class of theoretical injection pressures with fixed cycle fuel delivery g_c

$P_{o.p.}$ - over-plunger pressure

P_c - cylinder pressure

$p(\phi)$ - theoretical injection pressure

$\mathbf{q}$ - vector with limitative characteristics

$\mathbf{R}^m$ - m-dimensional Euclidean space

$\mathbf{R}^\nu_+$ - subspace of vectors from $\mathbf{R}^\nu$ with nonnegative components

$\mathbf{u}$ - vector with variable parameters

$V_{o.p.}$ - over plunger volume

V_c - cylinder swept

$\mathbf{W}$ - output space

$\mathbf{X}$ - state space

$\mathbf{\Pi}$ - parallelipiped from $\mathbf{R}^m$ contains of all admissible vectors with variable parameters

$\mu(\phi)$ - positive bounded measure

$\mu f_{n.h.}$ - effective area of nozzle holes

$\phi_{inj.b.}$ - crank angle of the injection beginning

$\phi_{inj.e.}$ - crank angle of the injection ending

$\phi_{m.p.e.}$ - crank angle of the maximal plunger elevation

$\phi_{s.b.}$ - plunger stroke beginning

$\omega(\phi)$ - weight function

REFERENCES

[1] Doyle, J.C., Francis, B.A., Tannenbaum, A.E., "Feedback Control Theory", Macmillan Publ. Co., New York, 1992.

[2] Arbib, M.A., Padulo, L., "System Theory", Hemisphere Publ. Corp., Washington, D.C., 1974.

[3] Belyi, S.V., Tsekanovskii, S.V., "Classes of Operator-valued R-functions and Their Realization by Conservative Systems", Soviet. Math. Dokl. no.3, v.44, pp. 692-696, 1992.

[4] Naylor, A.W., Sell, G.R., "Linear Operator Theory in Engineering and Science", Springer-Verlag, New-York-Heidelberg-Berlin, 1982.

[5] Grinshpan, A.Z., Romanov, S.A., "Improvement of Diesel Engine Perfomance through Fuel-Injection Equipment Optimization", SAE paper 911820, 1991.

[6] Sviridov, Yu.B., "Mixture Formation and Combustion in Diesel Engines" (in Russian), Mashinostroenie, Leningrad,1972.

[7] Grinshpan, A.Z., "The Use of Parameterization for Modeling Diesel Fuel Spray", SAE paper 921728 , 1992.